Virtual Reality vs Real Virtuality in Mathematics Teaching and Learning

Pavel Boytchev, *boytchev@fmi.uni-sofia.bg*
Faculty of Mathematics and Informatics, Sofia University, Bulgaria

Toni Chehlarova, *tchehlaroval@mail.bg*
Faculty of Mathematics and Informatics, Plovdiv University, Bulgaria

Evgenia Sendova, *jsendova@mit.edu*
Institute of mathematics and Informatics, BAS
Acad. G. Bonchev bl. 8
Sofia 1113

Abstract

The paper presents research activities carried out within DALEST Project (Developing an Active Learning Environment for the Learning of Stereometry). Some of the computer 3D applications based on Elica are considered together with possible educational activities in the context of nets of solids. The first impressions of the pilot experiment are presented from learners', educators'; and developer's perspective.

Keywords

Mathematics,3D geometry, teaching, learning, DALEST, Elica, Logo

PROBLEMS OF TRADITIONAL TEACHING 3D

Since *space is a fundamental category of thought* and a fundamental feature of our environment, spatial knowledge and spatial perception play crucial roles in even the most ordinary human problem solving [Report on The National Science Foundation Research Planning Workshop, 1997]. *Furthermore, the ability* to *think in pictures*, to perceive the visual world accurately, and recreate it in the mind or on paper, *called in short* spatial intelligence*, is identified as one of the seven different kinds of intelligence* [Gardner, 1993].

As discussed in [Grozdev, Chehlarova, 2005] the development of students' spatial imagination is still a problem which is not fully solved in Bulgarian Mathematics Education. According to the current syllabus some knowledge of prism, pyramid, cone, cylinder, ball and sphere should be formed during the compulsory education in 6^{th} grade. Superficial knowledge of 3D Geometry is learned in 12^{th} grade and as a profiling subject - in 11^{th} and 12^{th} grades. The new syllabus provides for cube and rectangular parallelepiped teaching in 5^{th} grade. But in the textbooks for the rest of the grades one wouldn't find even sporadic problems targeted at developing the students' spatial sense.

Providing students with possibilities to explore the properties of 3D objects in **appropriately developed computer applications** could improve the situation. Since children's initial representations of space are based on action an important feature of such applications is that they should be *action-based*, i.e. the students should be able to:

- *slide and rotate 3D objects creating dynamic images;*
- *combine 3D objects in more complex compositions;*
- *measure the basic 3D objects and study their properties and relationships;*
- *represent and solve problems using models of 3D objects.*

For students to *develop an appreciation of geometry as a means of describing the physical world* it is important to provide them with **carefully designed sequences of educational activities** within such computer applications.

The above-mentioned skills and competencies are identified by math educators as crucial for the development of the spatial sense in 5-8 graders [National Council of Teachers of Mathematics, 2000].

THE *DALEST* PROJECT

To enhance the 3D geometry understanding and spatial visualization skills of middle school students by working with dynamic visualization images in specialised computer environment has been the main goal of the project ***D****eveloping an* ***A****ctive* ***L****earning* ***E****nvironment for the Learning of* ***St****ereometry* [DALEST project: http://www.ucy.ac.cy/dalest/]. The theoretical background behind the design of the DALEST software has been considered in details in [Christou et al., 2006]. The applications we are referring to in this paper are a subset of DALEST applications developed in *Elica* - a modern Logo implementation providing tools for real-time 3D animation of user-programmable objects [Boytchev, *Elica*, http://www.elica.net]. In harmony with the constructionsim [Papert, 1991], These applications support educational activities that encourage the students to create and design mathematical objects.

The current phase of DALEST embraces the development of sequences of pedagogical scenarios expected to be *rich in content*, *visual in nature*, *related to the world* and *consonant with the need of the students for self-expression*.

After overviewing the DALEST applications in a nutshell we give possible pedagogical scenarios in an attempt to illustrate their teaching/learning potential.

DEVELOPING A NOVEL SET OF APPLICATIONS

The complete set of DALEST/Elica applications comprises of ten applications which can be grouped into several streams:

- *Cubix*, *Cubix Editor* and *Cubix Shadow* can be used
    - to build various structured composed of cubes
    - to measure or calculate their volume
    - to design the shadow they cast, or
    - just play meccano.
- *Potter's Wheel*, *Bottle Design* and *Math Wheel* applications support various activities related to designing rotational solids and calculating their surface and volume.
- *Stuffed Toys*, *Scissors* and *Origami Nets* are dedicated to exploration of nets – both folding nets to solids and unfolding solids to nets
- *Slider* is an application for studying the intersection of a solid and a plane.

The main goal of *Cubix* application is to find out (in various ways) the volume and the area of a structure built from unit cubes. There are three sets of problems inbuilt in this application - for beginners, for advanced and for *champions*. Every set has 9 tasks in ascending order of difficulty. When a cubical structure is displayed on the screen, the user can rotate it with the mouse. This is important when manipulating some cubical structures containing "gaps", "wholes" or "depressions" (Figure 1).

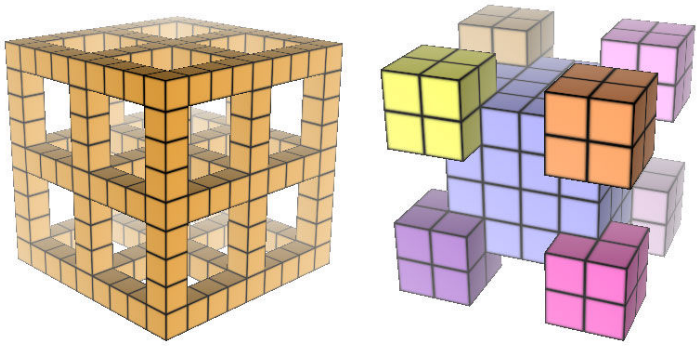

Figure 1: Two of the structures in the *Cubix* application

The *Slider* application creates an invisible 3D solid (a cube, a sphere, a cone, etc.) which can be cut through with a *magic* plane. In the *normal* mode only the intersection becomes visible. By moving and rotating the cutting plane students are supposed to analyze the changes in the intersection and find out the solid. Solids can have various orientations and can be made visible in the *Help me* mode - a feature which helps students understand how an object can "produce" various intersections – Figures 2 and 3.

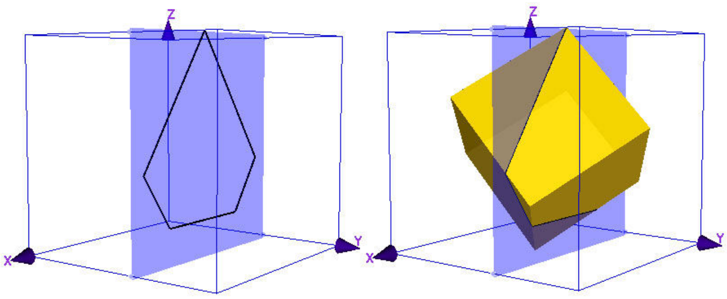

Figure 2: An *interesting* intersection of a cube shown in the *Slider* application

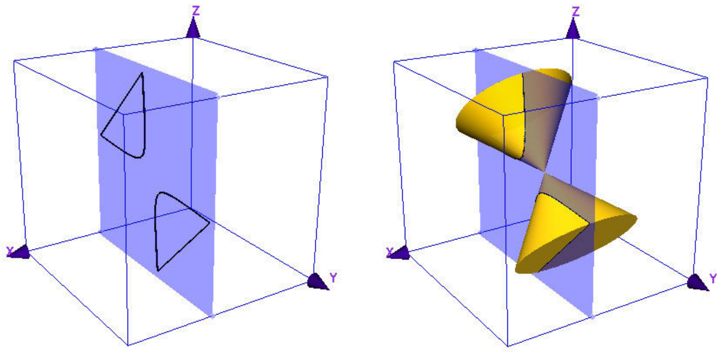

Figure 3: Exploring conical sections with the *Slider* application

In the *Scissors* application students have to solve the reversed problem of net folding - given a cube they must decide which edges to rip off, so that the remaining solid can be unfolded into a predefined net (Figure 4). Some of the problem sets provide not only cubes but other 6-faced objects like balls, octahedrons, ribbons (Figure 5). Users can rip along any of the edges. If they rip more edges than necessary, a net cannot be produced. If they rip fewer edges than necessary, the system will rip the rest (selecting arbitrary edges).

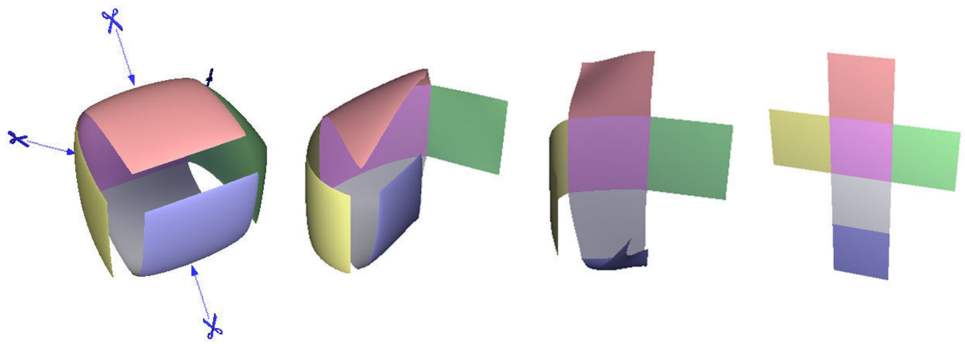

Figure 4: Ripping and unfolding a cube to a net with the *Scissors* application

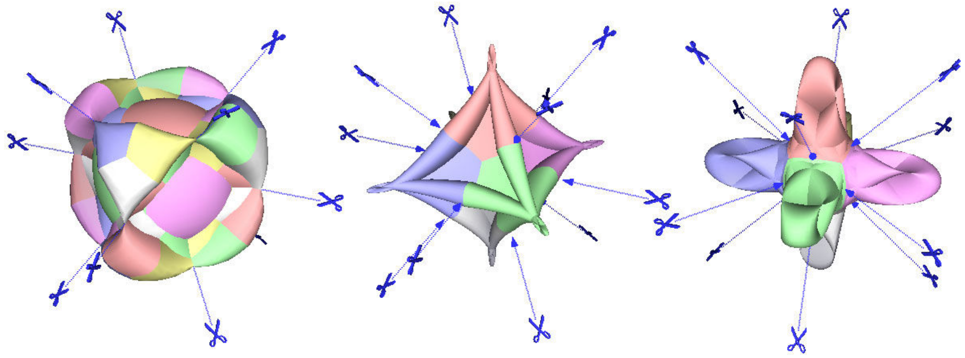

Figure 5: Objects from the *Ribbon set* are DALESTially equivalent to a cube

The *Bottle Design* application provides the user with a chain of connected segments. These segments define the profile of a bottle. There is an axis on the screen and the bottle profile is positioned on the right of the axis. When the user has edited the profile to his taste by moving the dots (Figure 6), the application rotates the object around the axis and produces a 3D rotational image. The *Bottle Design* enables the user to define bottles with specific volume and/or surface. A similar process is used in the *Potter's Wheel* and *Math Wheel* applications.

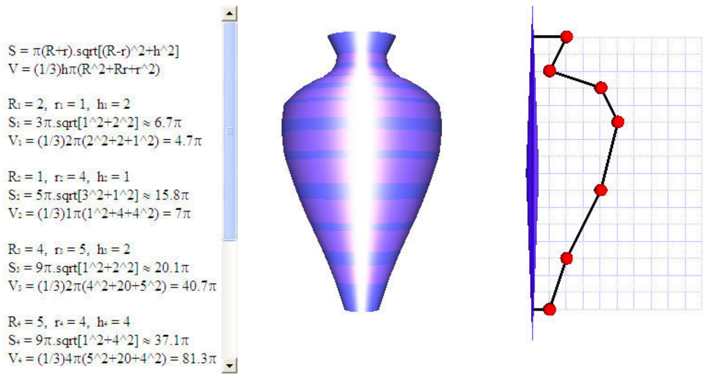

Figure 6: An amphora defined in the *Bottle Design* application

Origami Nets is the most complex application in the project. It allows the students to build freely foldable nets using a set of 2D shapes. The shapes can be attached (*glued*). Then each joint between two shapes can be folded at any angle. In this way students create various solids – cubes, cuboids, prisms, truncated prisms, cones, and others (Figure 7). An interesting challenge in the application is to construct more complex objects such as the ball in Figure 8.

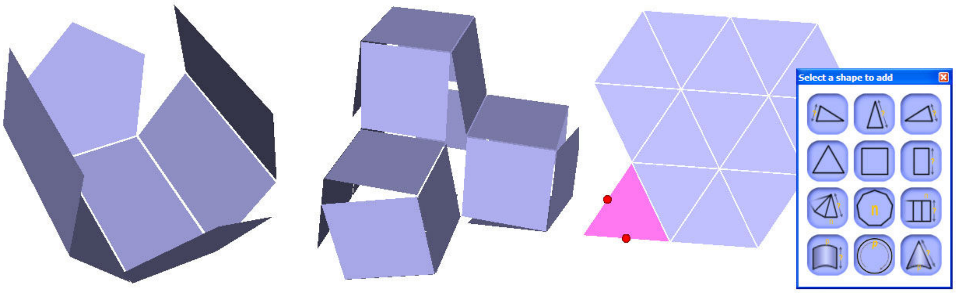

Figure 7: Using *Origami Net*s to construct a prism, a set of three cubes and to tessellate the plane with triangles

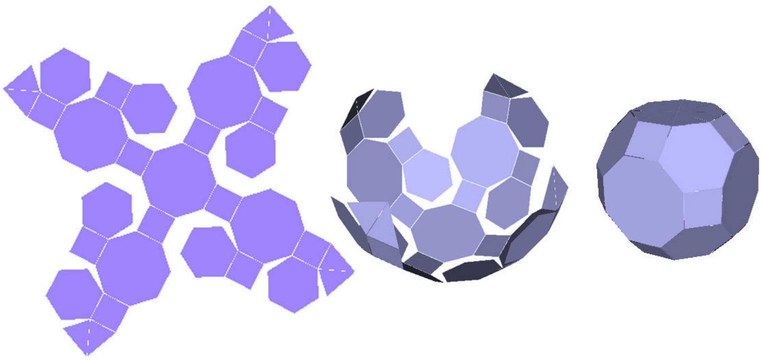

Figure 8: The *Origami Nets'* ball in unfolded, semi-folded and folded position

THE PILOT EXPERIMENT

The pilot experiment was designed so that we could estimate the time necessary for the kids to acquire a specific geometric unit by means of the DALEST software. Furthermore, we have to elaborate the directions for using the program and to establish some typical mistakes that would help us develop the chain of problems and the methodical approaches to solving them.

The didactical scenarios being developed in the frames of the DALEST project have the following structure:

- The Goal (formulated appropriately for the students)
- The Challenge (formulated as a motivating situation)
- A Chain of problems (with growing difficulty)
- Reflection (in the form of a letter to a friend)

Let us illustrate this with an example - in the scenario *Cuboid's net* the respective sections are as follows:

The goal: *In this activity you will identify and construct nets of cuboids. To check your conjecture you could use the program Dalest Origami nets.*

The challenge: Which one of the figures cannot be a net of the given cuboid-box (only folding but not cutting is aloud)? (Figure 9)

Figure 9

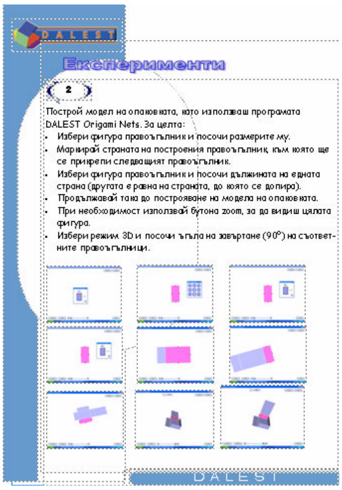

Figure 10

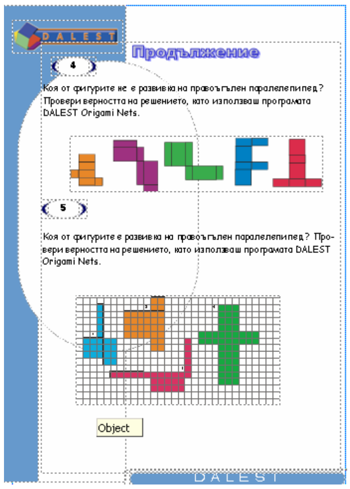

Figure 11

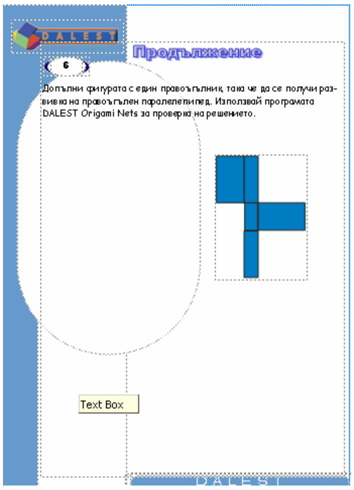

Figure 12

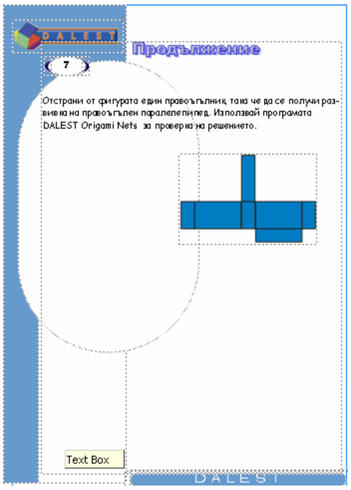

Figure 13

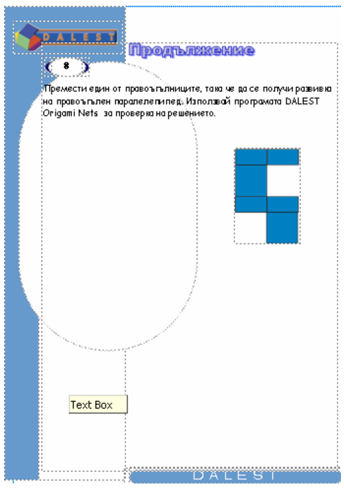

Figure 14

The Chain of problems includes:

- Directions how to use the program for constructing the net of a cuboid (Figure 10)
- First level problems which assure the required minimum for recognising developing a net of a cuboid (Figure 11)
- Second level problems related to manipulation with nets, to developing the 3D imagination of the kids, to enhancing their mathematical thinking. The problems in this activity deal with constructing a net by adding (Figure 12), removing (Figure 13), or shifting (Figure 14) a rectangle of a given figure

The Reflection – in this section the kids are expected to verbalize their actions while solving the problem in the form of a letter to a friend

FIRST IMPRESSIONS

Kids' perspective

The tradition had influenced the formulation of some of the problems, e.g. *construct a net and then a cube out of it.*

When constructing this problem though a child broke the usual approach of first making the net and only then folding it; rather, she applied a bottom-up approach by adding consecutive squares and folding them immediately.

What was impossible to implement with paper and scissors turned out to be the most natural way in the computer environment.
The option of folding a rectangle immediately after being constructed contributes to experimenting with different styles of constructing the net (step-by-step folding, folding after several foldings, and folding the net only after it has been fully constructed in the plane. Thus, the traditional construction of the net as a first task would be used as a further step in the mental process (requiring a higher level of abstract thinking).

Another observation was that when given the task of constructing a cuboid net by themselves the kids usually would make the nets in Figure 15:

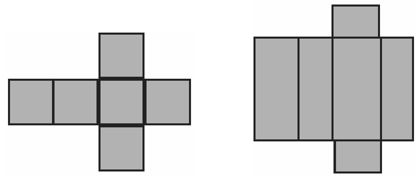

Figure 15: Kids' favorite nets

In order for them to manipulate with other types of nets and to develop their spacial thinking it turned out to be suitable to provide them with figures which could be turned into nets after concrete operations (such as adding, removing or shifting some of their parts).
The kids reacted emotionally when passing from 2D to 3D mode and vice versa (especially when they did this for a first time)

The teacher's impression that the students need a special recognition for their performance was confirmed by their words – orally and in written form.
Here follows a fragment from two such letters:

Koya, 11-year old

> *Not only do I know what a net is but also how to work with a program called Origami nets. You can construct a net and then pass in 3D – but to do this you need to tell in advance the angle of flopping of the squares. It is so much easier to construct nets on the computer screen since you only need to click and drag. If you like you could also print the net but then you would need to add small parts for sticking the sides together.*
> *What I liked the most was to figure out myself what to do. The first thing I made was a basket – almost like a cuboid but with two longer opposite sides. The second product of mine was something like a star – I selected a pentagon for a basis.*

Neda, 12-year-old

> *I learned how to make cuboids' nets. At first I found the program difficult but after I constructed a single net I realised that it was easy and interesting. I liked constructing nets of cuboids more than nets of cubes since it requires more thinking – in the case of a cube everything is clear – all the edges are equal. I like activities which make you think.*

As seen the kids are evaluating the software with respect to its user-friendliness and are discussing the advantages of working in a specialised computer environment – the possibility to check the solution of the problems by themselves. They are expressing their satisfaction of having mastered a tool which they can use for solving a whole class of problems. At the same time, they do not neglect the traditional work with paper models of solids and give ideas how the program could be improved for the purpose. Another interesting aspect is that the kids combine *debugging* with *degoaling* – when the object they are constructing is not what they have aimed at they are able to see another possibility.

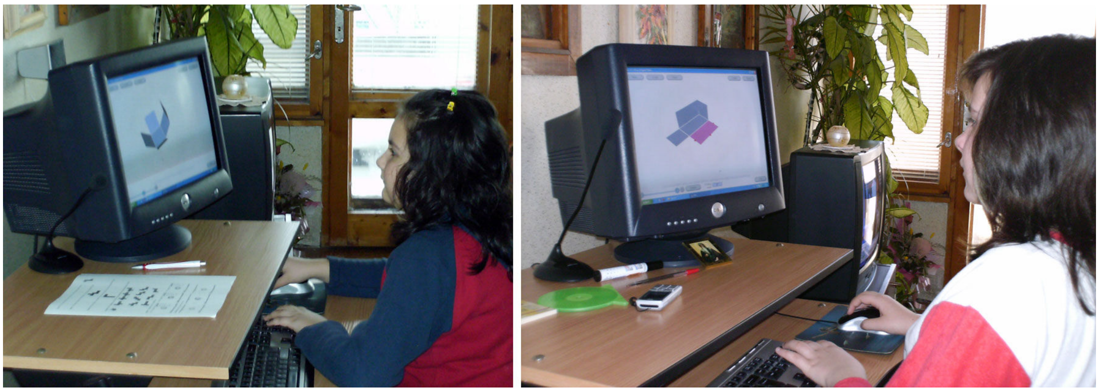

Figure 16: Students learning and playing with OrigamiNets

Teachers' perspective

The problems we have been envisaging with disseminating the first products of the DALEST project are related to the rigid class structure – a fixed number of classes on a certain topic and still limited access to computers in some schools. The math teachers will have to learn how to use the software and find time in the computer lab of the school. They still have to accept the idea of integrating the good traditions in teaching mathematics with using the new technology. Another possibility for implementing these programs is in the ICT classes which have been introduced from the current school year in 5th grade. Since the technical elements to be covered could be taught in various context the topic *Integrated activities* has been introduced by using two of the DALEST-Elica applications – *Cubix Editor* and *Origami Nets*. The first reactions of teachers implementing these programs are very encouraging. According to them the activities offered in the frames of DALEST are very motivating for the students, they shorten the time for preparing the math classes and allow for a self-evaluation of the students. The teachers share their satisfaction with providing the kids with meaningful and enjoyable hand-on activities with virtual objects. They emphasize the importance of the dynamic visualization of the movement from 2D to 3D representation of the objects such that all the intermediate states of the folding could be explored.

Developer's perspective

Creating educational software is not a novel activity by itself. However, the design and the implementation of software which can be used to achieve a concrete level of spatial thinking is not an easy task.

During the creation of DALEST/Elica application there were several challenges which the development team had to resolve:

- The first challenge was to determine what applications to design. There were some ideas of the didactical goals [Christou et al., 2006] to be achieved from mathematical perspective but the activities behind them were not decided upon (with a few exceptions). To make the situation more challenging the applications were supposed to be used in creative activities. Thus the functionality should allow the users to use the applications beyond any initial idea of the developers.
  A few brainstorming sessions were held addressing the issue of the basic concepts and expectations of the DALEST/Elica application. This design phase was crucial to the success of the project as it laid its foundations. As it was planned and as it actually happened later, the applications not only supported the preliminary set of activities, but also gave ideas about new type of activities quite disparate to the initial design. For example, the *Origami nets*, which was

supposed to be used in net-design and net-folding activities, could also be used in 2D activities, e.g. tessellating the plane by regular polygons.

- The second challenge in the software design was to make all applications multifunctional - usable as tools for solving and creating math problems, for experimenting with various solutions, and for creative explorations in various context, including arts.
- The third challenge was to build applications which were unique both individually and as a set designed for developing a specific complex of spatial thinking skills. At the time of starting the project there were many on-line and off-line applications for enhancing one's spatial intelligence. However, most of them were just different implementation of the same activity. While developing the DALEST/Elica applications, a special attention was paid to designing activities for which there was no software available. This imposed a specific requirement to the design. As a result, many of the developed applications can be used for solving problems that are reversed of some well known ones. For example, the *Scissors application* provides tools for solving the reversed problem of the net folding, namely how to rip off the edges of a cube in order to get a given net. The initial observations showed that such reversed problems were not so easy to solve. The main reason for this was that users have been exposed in the past to seemingly similar set of problems, but the reversed problem required them to step aside from the *beaten path*.
- The forth challenge was related to the primary design of the applications. The activities, which they were supposed to support, had to be hard or even impossible to present on paper. The main purpose of this requirement was to ensure that the applications were not just electronic version of the traditional textbooks, but would have an added value. One of the ways to comply with this requirement was to suggest activities which needed a 3D object to be rotated and viewed from different perspectives.
- The fifth challenge in the development of DALEST/Elica applications was to make them dynamic and attractive for students (of different age). The nowadays society is bombarded by a massive stream of visual information – TV ads, movies, video games. As a result the users have, by default, extremely high expectations about the visual appearance of the software. To implement this in a set of educational application, it was needed to arm each module with features such as 3D visualization and interactive real-time control of 3D objects. This posed another interesting sub-challenge – in order to carry out the didactical scenarios in a 3D virtual environment the users had to use a simple and intuitive interface (a few buttons and a mouse).

CONCLUSION

Although the educational activities within DALEST are still to be implemented on a larger scale the first reactions of teachers and students alike are very encouraging. The impression is that now that the students are provided not only with virtual reality but with real virtuality they will rediscover the mathematics as an experimental science in which playing with ideas is exciting.

Acknowledgments

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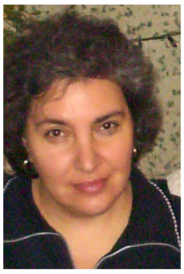
Biographies

Pavel Boytchev is an associated professor at Faculty of Mathematics and Informatics, Sofia University. Author of Elica and a developer of educational software. Designed and taught courses "Computer Graphics" and "System and Environments for Electronic Education".



Tony Chehlarova is assistant professor at Faculty of Mathematics and Informatics, Plovdiv University. She is involved in mathematics education by teaching students from 1st to 12th grade and training teachers-to-be. She delivers courses in Mathematical Didactics, Methods in Teaching Mathematics, Methodology of the Pedagogical research, methods for Problem Solving, Implementing ICT in Education.

Evgenia (Jenny) Sendova is a senior researcher at the institute of Mathematics and Informatics at the Bulgarian Academy of Sciences. Her interests are in using informatics and ICT for teaching mathematics, languages, music, science and arts. She has been working on developing Logo microworlds and models for integrating learning and creative processes. For more than ten years Jenny has been involved in working with high school students highly motivated in studying mathematics and science.