

Re-experiencing Engineering Inventions within a Modern Virtual Environment

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Abstract. This paper describes a personal journey in the world of Mathematics and Mechanics and the actual process of reinventing devices with focus on their mathematical and mechanical properties. A set of important accompanying skills have been identified along the path of constructing these devices.

Keywords: virtual devices, mathematical curves, mechanics, linkage

1 The mechanics of trigonometric functions

One of the most notable features of the modern life is that many things are converted into virtual things. We could pay with virtual money, we could enroll in virtual distant courses, we could visit virtually a tourist destination and we can have a virtual talk with virtual friends.

(Un)fortunately we have our temptation to make and to construct drifted away towards virtual artifacts. It is unfortunately, because we do not use our hands – we cannot touch, smell and taste our work. Nevertheless, it is also fortunately, because the virtualization revives the almost lost joy of constructing things.

Mechanical devices are one of the most interesting machines built by humans that are not as sophisticated as electronic devices, but can provide a perfect possibility to see and to understand how a set of components work together in an ideal synchronization.

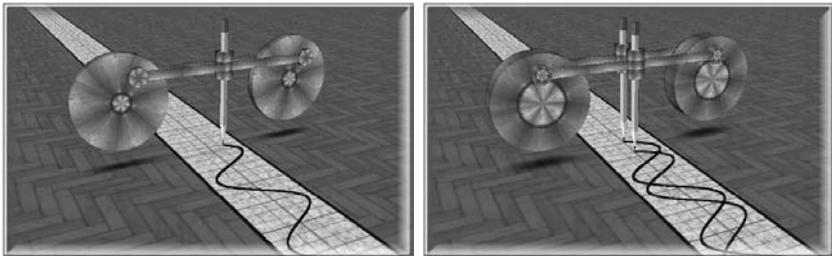


Fig. 1. Devices drawing sine (left) and sine/cosine graphs (right).

After a talk with a virtual friend, it was decided to make a model of a device that draws the graph of the sine function using Elica (www.elica.net) as a development environment. Elica uses the programming language Logo, specifically designed for education. Additionally, Elica supports 3D graphics – the basis of virtual reality [1].

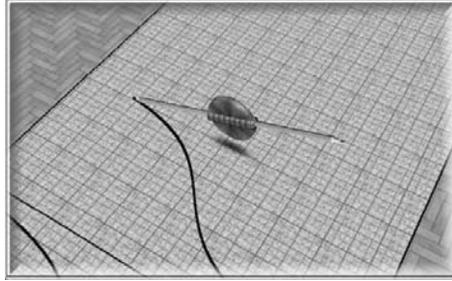


Fig. 2. Modeling tangent with a sliding pencil.

The initial idea is to have a pen attached to a rotating disk. However, most likely such device will draw a cycloid. The solution is to use orthogonal orientation as shown in Fig 1 (left). The sine device leads us to a new problem – cosine has the same shape. Is it possible to build a device that draws both graphs at the same time?

Instead of shifting joints “along” the angle of rotation, it is possible to shift them along time. Then we can convert this shift into a shift in space by moving the pencils away of each other – Fig 1 (right). Depending on the pedagogical arrangements, the constructions in Fig 1 can utilize or develop an important skill *to have an active knowledge of the functions’ properties*. In this context active means that the student can pick and use the right property in the right time.

Sine and cosine are mathematically bound to the tangent. It has nice geometrical and algebraic explanations, though none of them is practical if we want to build a mechanical device. There is one problem – the infinity. The tangent function can produce infinitely large and small values. To make the situation worse, the graph jumps abruptly from positive infinity to negative infinity [2].

An interesting solution is to use a double-sided pencil, which slides forward and backward. Thus, the gravity does all the work – it slides the pencil in the right direction. When the pencil is horizontal, its tips point towards both infinities at the same time. This solution relates to the next important skill, which is *to transform the impossible things into possible*.

2 The cycloid family

A device similar to the sine device can draw all forms of cycloids. Fig 3 (top) shows a device drawing a prolate cycloid and a modification for drawing nephroid [3].

A specific challenge is how to roll over the pencil. One of the possible solutions is to make small niches at every place where the pencil is between the two circles. This approach expects some mathematical study ahead of making the model, because the number of niches and their positions must be pre-calculated.

This solution relies on the skill of *discovering various tricks that resolve the conflicts between the ideal mathematics and the physical world*. Unfortunately, this skill is very hard to teach and it is not possible to provide a prescription for it.

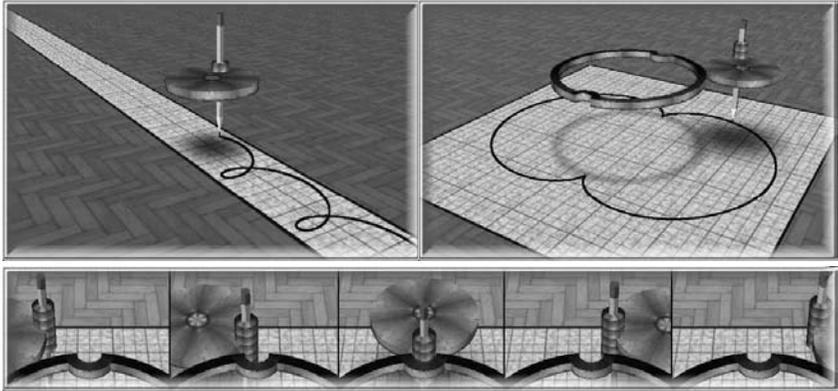


Fig. 3. Prolate cycloid (left), nephroid (right) and niches.

3 Linears and circulars – interchanged

Cycloids, epicycloids and hypocycloids are families of quite rounded curves. Someone may intuitively expect that rounded objects are suitable for generating round curves, but would it be possible to construct straight lines? As shown in Fig 4 (left), a hypocycloid may become a segment for a specific ratio of the radii. It is a nice mathematical exercise to prove that the curve is really a fragment of a line.

We could also ask the question how to make a round object by using only straight linear objects and linear movements. The bars of the device in Fig 4 (right) perform only linear movement along perpendicular rails and draws an ellipse. It is even possible to find a way to flatten the ellipse completely into a segment as shown in Fig 5 (left).

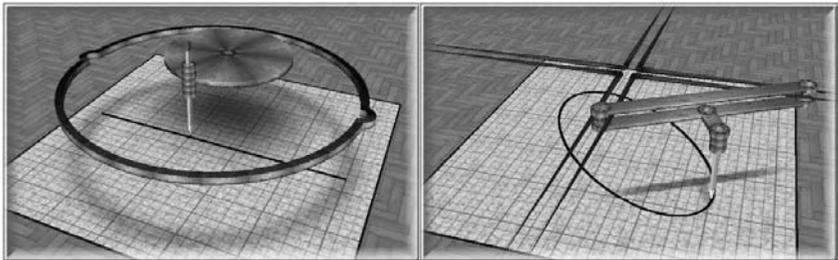


Fig. 4. A segment from a hypocycloid and an ellipse from a straight line.

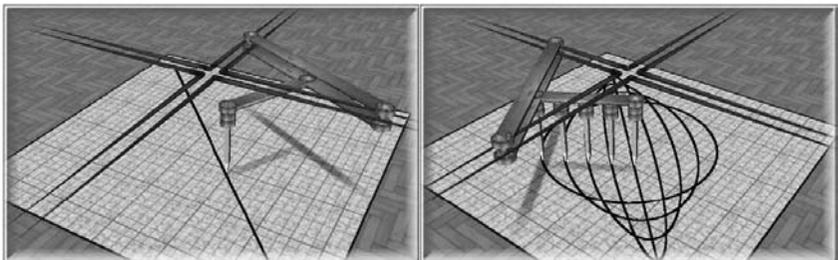


Fig. 5. Segment (left) and the family of curves (right).

A mechanism combining the drawing of a circle, ellipses and a segment is displayed in Fig 5 (right) and is an illustration of the transformation from a circle to a segment via different phases of ellipses. The construction of this combined device relates to the skill of *finding the common features of distinct models and converting them into special cases of a single united global model*.

4 One for all

Circles and ellipses are members of the family of conic sections. A plane crossing a cone generates intersection, which could be a circle, an ellipse, a parabola or a hyperbola [5]. Is there a single device can generate any of the conical sections? The answer is “yes” if we assume that the paper is the plane, and that a pencil rolls over the surface of a cone. Apart from rolling, the pencil slides up and down the cone to ensure that it reaches the paper and leaves a trace.

Fig 6 demonstrates a disk with tilted pencil that is capable of drawing all four conical figures. While the disk is rotating, the tilted pencil shapes an imaginary cone. By tilting the disk and the holder, we can control which of the figure we want to produce. For a circle, we need a horizontal disk. A slight change in orientation will generate an ellipse. When the pencil becomes horizontal at its highest position, then we have a parabola (i.e. the pencil is “trying” to write in the infinity). A hyperbola is generated when a double-tipped pencil draws from both sides of the disk.

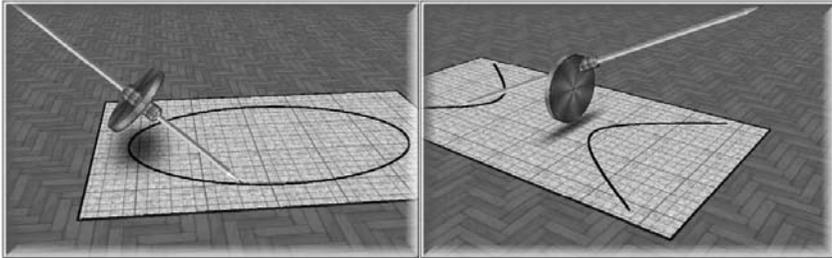


Fig. 6. Ellipse (left) and hyperbola (right).

The design of a device that can draw various conical curves requires not only the skill to unite various cases into one, but also the skill *to capture, explore and interpret the fundamental characteristics*.

5 More complex models

Lemniscate of Bernoulli. A linkage may produce the famous pendant ribbon curved called lemniscates of Bernoulli [6] – Fig 7 (left). The pencil is in the middle of the central bar. What will happen if it is placed elsewhere? The curve’s shape must be something between the shape of 8 and a circle. How would it look like? A modification of the program shows how one of the loops gradually shrinks into a cusp and then melts into the circle emerging from the other loop. The case with the cusp is presented in Fig 7 (right).

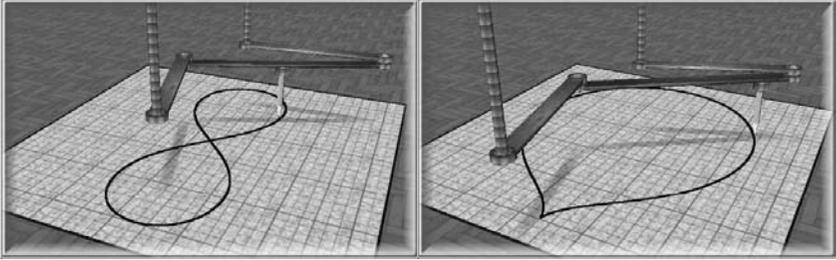


Fig. 7. Lemniscate of Bernoulli (left) and lemniscate drop (right).

The hyperboring (sic) cosine. Initially the graph of the hyperbolic cosine function was considered by the author as a boring curve, not worth implementing as a device. However, it has been found that the curve has many names – catenary, alysoid, funicular, chainette. This is an indication that the curve is something special. In reality, the hyperbolic cosine function is very popular – its graph is the shape of the freely hanging chain or high-voltage cables, the silhouette of an igloo, and the arks in a cathedral. Many of the suspension bridges utilize the properties of the hyperbolic cosine, because it allows a perfect distribution of forces.

The designs of suspension bridges are shown in Fig 8. The lengths of the wires in both models in are the same. The top ends of the left bridge forms a catenary. The same wires can be focused into one point as shown in the right bridge.

A device that would use variable length bars is hard to design, but is it possible to use fixed-length bars? The positive answer needs some mathematical experiments. If we consider the scheme of the right bridge as a side view of a cone, then all bars would physically have the same lengths and the hyperbolic cosine will emerge in their shadows (or orthogonal projections). Adding some more supplementary elements leads to an interesting solution of a catenary drawing device – Fig 9 (left).

The central point of the upper beam is moving like an upwards pendulum and traverses the semiperimeter of the base of an imaginary cone in Fig 9 (right).

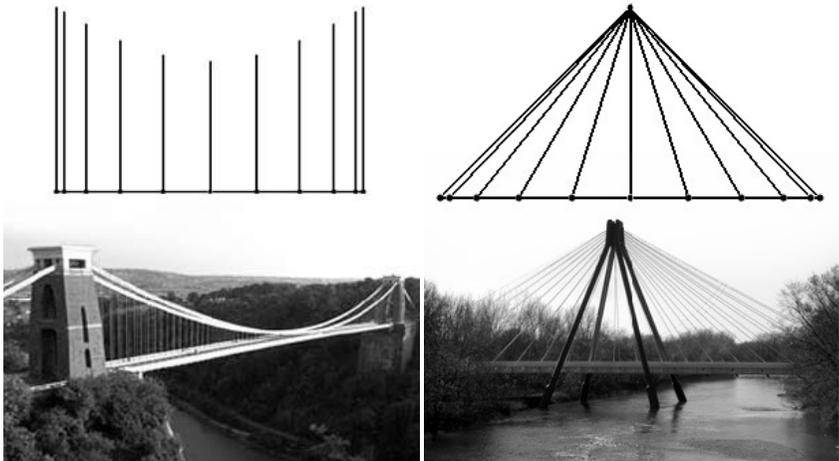


Fig. 8. Suspension bridges.

If this central point were connected with the cone vertex with a bar, then the length of its shadow would be the same as the length of the shadows of the two bars that hold the pencil. These shadows are the same as the wires in the suspension bridges in Fig 8.

The hyperbolic cosine device demonstrates one very math-aware skill: sometimes a person needs *to build virtual constructions in order to cross the problem and reach the solution on the other side*.

6 The square puzzle

A dozen of different devices are shown in this paper so far, some of them are simple, others – not so simple. It is not feasible to provide device for all known curves, but it would be nice to give an idea of how we can approach the construction of any curve including polygons.

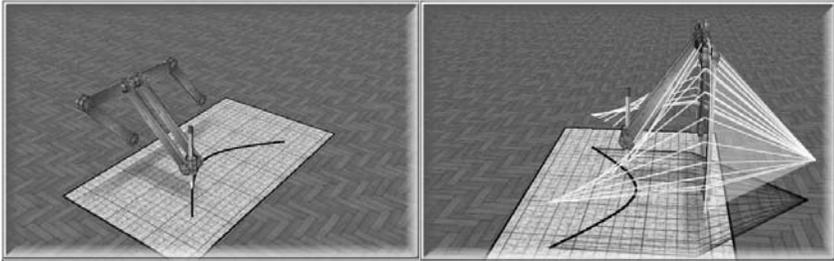


Fig. 9. A device drawing a hyperbolic cosine (left) and the imaginary bridges (right).

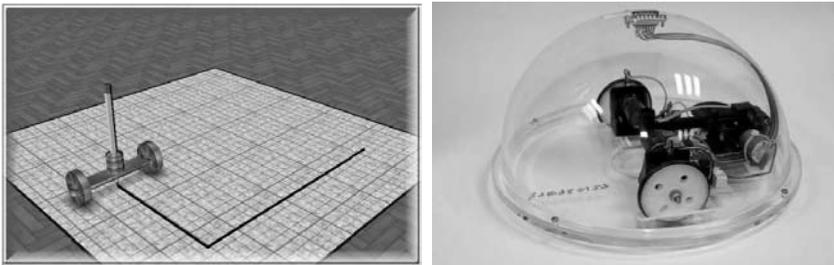


Fig. 10. Virtual model (left) of a floor turtle and a photo of a real turtle-robot¹ (right).

Imagine a square – this is quite common curve. The problem is how to draw it with a device that has two disks and a bar. Although this looks impossible, as promised in the title of the paper, we will re-experience the engineering inventions.

Fig 10 shows the virtual skeleton and a photo of a floor turtle. This is a wheelbased robot, which can go forward and backward, as well as turn left or right. By controlling the rotation of the wheels, we can make the robot draw any curve that can be expressed algorithmically with a sequence of Turtle graphics commands. We used a programming language (Elica Logo) to build devices

¹ National Education Network Gallery, <http://gallery.nen.gov.uk/image79109.html>.

drawing curves in order to evolve to the ultimate drawing programmable device – the turtle. A similar evolution occurred in the history of mankind, mechanical devices from the past are now being replaced by electronic, requiring a new skill: *algorithmic thinking and programming!*

7 Conclusion and Ideas

The presented journey in the world of designing mechanical devices for drawing mathematical curves reveals the coherent linkage between the world of Mathematics and the real life. Whatever we design and construct, we need some way of representing our ideas. Whether we use real tools and materials, or just programs and virtual worlds, we still need to spend some time in researching, exploring and constructing.

The design and development of these devices can be used in a variety of educational situations. They can be used in Mathematical classes for exploring curves and their properties, in Engineering classes for designing linkage devices, in Computer science or Computer graphics classes for programming 3D virtual microworlds ... and finally they can be used to develop a set of multifunction interdisciplinary skills:

- Resolving theory-practice conflicts and transforming the impossible into possible
- Identifying, interpreting and exploring fundamental characteristics of objects/phenomena
- Simplification, unification and optimization
- Cross-disciplinary know-how transfer
- Algorithmic thinking

Additionally, another specific skill is silently embroidered throughout the whole journey. This skill is the devising of problems which solutions push us further along the path of constructionism.

A short 30-seconds long animation of each device is uploaded on the YouTube in the ElicaTeam profile. The future plans for these devices span over several directions. Obviously more devices will be added to the list, but it will be important to build a constructor's microworld where the elements of the devices are available as programmable objects. Thus, even novice Logo users will be able to experiment with their own custom-designed devices. From educational point of view, it would be nice if the topics discussed in this paper were converted into learning and teaching activities. The design, the reasoning and the implementation of each device is worth a whole lesson by itself. A real in-class application of the ideas in the paper is still something essential to be done in the future, because this will provide information about how applicable this approach is, as well as it will be a source of new ideas and developments.

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